Helicity asymmetry for proton emission from polarized electrons in the eikonal regime

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Abstract

The nuclear response to longitudinally polarized electrons, detected in coincidence with out-of-plane high-energy protons, is discussed in a simple model where the ejectile wave function is approximated as a plane wave with a complex wave vector. This choice is equivalent to solve the problem of Final-State Interactions (FSI) in homogeneous nuclear matter, as the residual nucleus can be described to a first approximation when dealing with very fast emitted protons. The main advantage of the present method is that in the framework of the Distorted-Wave Impulse Approximation (DWIA) at the one-photon exchange level it allows for an analytical derivation of all the components of the nuclear response, including the socalled fifth structure function f'_{01}, which is very sensitive to FSI. The imaginary part of the complex wave vector produces purely geometrical FSI effects and, consequently, breaks the symmetry of the cross section with respect to the incoming electron helicity. Inspection of every single contribution in the analytical formulae, here considered up to the fourth order in the nonrelativistic reduction in powers of the in-

verse nucleon mass, allows for a detailed study of the role of each elementary reaction mechanism. In particular, cancellations among the leading contributions determine the very small absolute size of f'_{01} and produce a nontrivial asymptotic scaling of the related helicity asymmetry for large values of the momentum transfer.

25.30.Rw, 25.30.Dh, 11.80.Fv, 24.70.+s .

I. INTRODUCTION

Nuclear reactions induced by electromagnetic probes are well known to represent a powerful tool to investigate the properties of nuclear structure, because the whole target volume can be explored and the electromagnetic interaction with the external probe is well described by the theory of Quantum Electrodynamics (QED) [1–3].

In the case of electron scattering, the additional ability of independently varying energy and momentum transferred to the target, as well as high-quality beams with large duty factors delivered by modern electron accelerators, allows for a detailed mapping of the nuclear response over very different kinematical conditions [4].

The power of this tool can be better exploited by requiring exclusive measurements like in the case of $(e, e'p)$ reactions, where the proton is detected in coincidence with the final electron and, when possible, for a specific energy range corresponding to a well-defined quantum state of the residual nucleus. The richness of the structure of the theoretical cross section indicates that under suitable kinematical conditions it is possible to disentangle observables which are selectively sensitive to certain ingredients of the theoretical model [4,5].

Additionally, if polarization observables can be measured, it is in principle possible to determine all the independent scattering amplitudes [6]. However, this experimentally formidable goal is far from being achieved. More simply, if just the electron beam is polarized, it is possible to isolate the socalled fifth structure function, which is generated by the interference between two or more reaction channels with different competing phases [2,7]. Measurements of the corresponding cross section asymmetry with respect to electron helicity can be used to explore the interfering reaction amplitudes, most of which are usually very small and otherwise hidden in the unpolarized case [8]. In electroproduction of pions the fifth structure function may provide a key observable for the isolation of the resonating channel in the $N \to \Delta$ transition, corresponding to a quadrupole deformed excitation of $\Delta^+(1232)$ [9]. Also in inclusive electron scattering from polarized targets the helicity asymmetry is needed to access observables like the neutron form factor [10] or the spin-dependent

nucleon structure functions [11].

Here, the completely exclusive quasielastic $(\vec{e}, e'p)$ reaction on nuclear targets will be considered. The fifth structure function is then given by the interference between the direct knockout and the rescattering channels and is therefore highly sensitive to Final State Interactions (FSI) between the outgoing proton and the residual nucleus. This issue has become crucial at the new (CEBAF) and planned (ELFE [12]) high-energy electron accelerators, where experiments with electromagnetic probes at momentum transfer beyond 1 GeV/c (in particular $(e, e'p)$ reactions [13–15]) are expected to shed some light on exotic phenomena predicted by the perturbative Quantum Chromodynamics (pQCD), such as for example the Color Transparency (CT). In fact, the experimental signal is predicted to be very small in this energy domain and a reliable model for FSI is needed to verify the CT prediction [16–18].

In the present literature the most popular and widely adopted approach is the Glauber method [19], which has a long well-established tradition of successfull results in the field of high-energy proton-nucleus elastic scattering [20]. Despite the high energy regime to which it is applied, this method is developed in a completely nonrelativistic formalism within the eikonal approximation. Because for a fastly moving object the nuclear density can be considered roughly constant inside all the nuclear volume but the small part corresponding to the surface, the eikonal wave function of the ejectile can be approximated by a damped plane wave, which corresponds to the solution of a Schrödinger equation inside homogeneous nuclear matter. In fact, in a previous paper [21] the angular distribution of emitted protons with outgoing energy beyond the inelastic threshold and with initially bound momentum below the Fermi surface has been shown to be well reproduced by actually assuming a plane wave for the final nucleon state with an additional damping. Therefore, in the following the scattering wave function will be represented as a plane wave with a complex wave vector, whose imaginary part produces a constant damping. Consequently, in the framework of the Distorted-Wave Impulse Approximation (DWIA) at the one-photon exchange level analytical formulae can be derived for all the components of the nuclear response. Moreover, the

presence of a damping in the outgoing plane wave produces a FSI effect of purely geometrical nature and generates an asymmetry of the cross section with respect to the helicity of the electron beam.

After a short review on the general formalism (Sec. II), analytical formulae for the fifth structure function and the helicity asymmetry will be deduced in Sec. III. Results for typical kinematics above the inelastic threshold will then be discussed with specific emphasys on the asymptotic behaviour for large momentum transfer (Sec. IV). Finally, some conclusions will be outlined (Sec. V).

II. GENERAL FORMALISM

The differential cross section for the scattering of a polarized electron, with helicity h and initial (final) momentum $\mathbf{p}_{\mathrm{e}}$ ($\mathbf{p}'_{\mathrm{e}}$), off a nuclear target from which a nucleon is ejected with final momentum $\mathbf{p}'$, can be written in the one-photon exchange approximation as [4]

$$\begin{aligned}\frac{\mathrm{d}\sigma_h}{\mathrm{d}\mathbf{p}'_{\mathrm{e}}\mathrm{d}\mathbf{p}'} &= \frac{e^4}{8\pi^2}\frac{1}{Q^4 p_{\mathrm{e}} p'_{\mathrm{e}}}\left(\rho_{00}f_{00} + \rho_{11}f_{11} + \rho_{01}f_{01}\cos\alpha + \rho_{1-1}f_{1-1}\cos 2\alpha\right. \\ &\qquad\qquad\qquad \left. + h\rho'_{01}f'_{01}\sin\alpha\right) \\ &\equiv \Sigma + h\Delta \ , \end{aligned} \tag{1}$$

where α is the out-of-plane angle (see Fig. 1), $Q^2 = \mathbf{q}^2 - \omega^2$ and $\mathbf{q} = \mathbf{p}_{\mathrm{e}} - \mathbf{p}'_{\mathrm{e}}$, $\omega = p_{\mathrm{e}} - p'_{\mathrm{e}}$ are the momentum and energy transferred to the target nucleus, respectively. The cross section is explicitely separated into a helicity dependent term Δ and the helicity independent unpolarized cross section Σ. The tensor $\rho_{\lambda\lambda'}$ depends only on the properties of the electromagnetic vertex and its components are completely determined by QED [2,3]. The tensor $f_{\lambda\lambda'}$ contains all the information about the target, in particular about the longitudinal ($\lambda = 0$) and transverse ($\lambda = \pm 1$) components of the nuclear response with respect to the polarization of the virtual photon exchanged. It is given as a bilinear product of matrix elements of the different helicity components of the nuclear current, which describe the transition from the initial to the final hadronic states. Therefore, in principle it involves

many-body matrix elements. However, in the projection operator approach [22] and within the framework of DWIA it is possible to project out of the total Hilbert space a suitable channel where the matrix elements, usually called spectroscopic amplitudes, are written in a one-body representation as [4]

$$J^{\lambda}_{nljm_l s's}(\mathbf{q}) = \int \mathrm{d}\mathbf{r}\mathrm{d}\sigma \ \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}} \chi^{(-)\,*}_{s'}(\mathbf{r},\sigma) \ \hat{J}_{\lambda}(\mathbf{q},\mathbf{r},\sigma) \ \phi_{nljm_l s}(\mathbf{r},\sigma) \ . \tag{2}$$

They describe the knockout of a nucleon leaving a hole with quantum numbers $(nljm_l s)$ and propagating across the residual nucleus with the scattering wave function $\chi^{(-)}_{s'}$, s' being the final (detected) spin. The normalization of the bound state ϕ is the spectroscopic factor, which measures the probability that the residual nucleus can indeed be considered as a pure hole generated in the target nucleus by the knockout process. The boundary conditions for the scattering wave $\chi^{(-)}_{s'}$ are such that each incoming partial wave coincides asymptotically with the corresponding component of the plane wave associated to the outgoing proton momentum $\mathbf{p}'$ and spin s'.

The amount of $(e,e'p)$ data presently available can be explained within the DWIA by adopting for $\phi, \chi^{(-)}$ the solutions of eigenvalue problems with phenomenological, single-particle, local, energy- and spin-dependent potentials of the Woods-Saxon type [4]. The current operator $\hat{J}_{\lambda}$ is usually approximated by a nonrelativistic expansion in powers of the inverse nucleon mass by means of the Foldy-Wouthuysen canonical transformation [23]. In terms of its controvariant coordinates $\hat{J}^{\mu} = (\rho, \mathbf{J})$, it is given up to fourth order by [24]

$$\begin{aligned}
\rho^{(0)} &= F_1 \ , \qquad\qquad \rho^{(1)} = 0 \ , \\
\rho^{(2)} &= -\frac{1}{8m^2}(F_1 + 2\kappa F_2)\left(Q^2 + 2\mathrm{i}\sigma\cdot\mathbf{p}\times\mathbf{q}\right) \ , \qquad \rho^{(3)} = 0 \ , \\
\rho^{(4)} &= \frac{1}{16m^4}\left(\frac{F_1}{24} + \kappa F_2\right)\left[(\mathbf{p}+\mathbf{q})^2 + \mathbf{p}^2\right]\left(Q^2 + 2\mathrm{i}\sigma\cdot\mathbf{p}\times\mathbf{q}\right) + \frac{15}{384m^4}F_1\left[(2\mathbf{p}+\mathbf{q})\cdot\mathbf{q}\right]^2 \\
&\quad + \frac{17}{384m^4}F_1\left[(\mathbf{p}+\mathbf{q})^2 + \mathbf{p}^2\right]\left(q^2 + 2\mathrm{i}\sigma\cdot\mathbf{p}\times\mathbf{q}\right) \ ,
\end{aligned}$$

$$\begin{aligned}
\mathbf{J}^{(0)} &= 0 \ , \qquad\qquad \mathbf{J}^{(1)}\frac{F_1}{2m}(2\mathbf{p}+\mathbf{q}) + \frac{F_1 + \kappa F_2}{2m}\mathrm{i}\sigma\times\mathbf{q} \ , \\
\mathbf{J}^{(2)} &= -\frac{F_1 + 2\kappa F_2}{8m^2}\mathrm{i}\omega\sigma\times(2\mathbf{p}+\mathbf{q}) \ ,
\end{aligned}$$

$$
\begin{aligned}
\mathbf{J}^{(3)} &= -\frac{1}{16m^3}\left\{2F_1\left[(\mathbf{p}+\mathbf{q})^2+\mathbf{p}^2\right]+\kappa F_2 Q^2\right\} - \frac{\kappa F_2}{8m^3}\mathrm{i}\sigma\cdot(2\mathbf{p}+\mathbf{q})\mathbf{p}\times\mathbf{q}\,, \\
\mathbf{J}^{(4)} &= \frac{1}{16m^4}\left(\frac{F_1}{24}+\kappa F_2\right)\mathrm{i}\omega\sigma\times(2\mathbf{p}+\mathbf{q})\left[(\mathbf{p}+\mathbf{q})^2+\mathbf{p}^2\right] \\
&\quad + \frac{17}{384m^4}F_1\omega\left[(\mathbf{p}+\mathbf{q})^2+\mathbf{p}^2\right]\left[\mathbf{q}+\mathrm{i}\sigma\times(2\mathbf{p}+\mathbf{q})\right] \\
&\quad + \frac{15}{384m^4}F_1\omega(2\mathbf{p}+\mathbf{q})\cdot\mathbf{q}\,(2\mathbf{p}+\mathbf{q}+\mathrm{i}\sigma\times\mathbf{q})\ , \qquad (3)
\end{aligned}
$$

where m, κ are the proton mass and anomalous magnetic moment, respectively, and F_1, F_2 are the Dirac, Pauli proton form factors

$$
\begin{aligned}
F_1(Q^2) &= \left(1+\frac{Q^2}{4m^2}\right)^{-1}\left[G_{\mathrm{E}}(Q^2)+\frac{Q^2}{4m^2}G_{\mathrm{M}}(Q^2)\right], \\
\kappa F_2(Q^2) &= \left(1+\frac{Q^2}{4m^2}\right)^{-1}\left[G_{\mathrm{M}}(Q^2)-G_{\mathrm{E}}(Q^2)\right] \qquad (4)
\end{aligned}
$$

with $G_{\mathrm{E}}, G_{\mathrm{M}}$ parametrized as in Ref. [25].

If the electron beam is polarized, the fifth structure function f'_{01} enters the cross section as described in Eq. (1) and is given by the following bilinear product of scattering amplitudes [4]

$$
f'_{01} = \frac{2q}{Q}\sum_{s'm_l s\overline{m_l s}}\left(l\tfrac{1}{2}m_l s|jm_j\right)\left(l\tfrac{1}{2}\overline{m_l s}|jm_j\right)\ \mathrm{Im}\{J^{0}_{nljm_l s's}J^{1\ *}_{nlj\overline{m_l}s'\overline{s}} - J^{0}_{nljm_l s's}J^{-1\ *}_{nlj\overline{m_l}s'\overline{s}}\}\,. \qquad (5)
$$

A necessary condition for having an imaginary component of the interfering longitudinal-transverse response is the presence of at least two competing reaction amplitudes with different phases [7]. As already mentioned, in quasielastic nucleon knockout the two dominant channels are the direct emission and the rescattering. Therefore, in the absence of any FSI, the socalled Plane-Wave Impulse Approximation (PWIA), the f'_{01} identically vanishes because the absence of any rescattering makes the bilinear products in Eq. (5) purely real and symmetric around $\mathbf{q}$. Therefore, the f'_{01} provides a suitable observable to monitor the rescattering processes in $(\vec{e}, e'p)$ reactions and may permit a much higher precision in constraining the models of FSI by isolating important and otherwise inaccessible reaction amplitudes. However, this is possible only at the cost of measuring the reaction products out of the scattering plane.

From the experimental point of view it is more advantageous to isolate the helicity dependent term in Eq. (1), Δ, which is proportional to f'_{01}, by measuring the asymmetry

$$A = \frac{\mathrm{d}\sigma_+ - \mathrm{d}\sigma_-}{\mathrm{d}\sigma_+ + \mathrm{d}\sigma_-} = \frac{\Delta}{\Sigma} , \tag{6}$$

because the systematic uncertainties in all the spectrometer efficiencies, target thickness, charge collection, cancel in the ratio [8]. FSI can break the symmetry when flipping the electron helicity h or, equivalently, when reaction products are scattered above or below the scattering plane for a given h. The different path followed on the way out of the nucleus makes outgoing protons have different rescatterings with the residual and, consequently, produces a phase difference with respect to the channel where they are knocked out directly as free particles. Therefore, at variance with what happens in the case of $(\vec{e}, e')$ where the asymmetry arises from a parity violating interaction, here A can be generated for simple geometrical arguments just by the modification of the plane wave of the final protons.

III. A SIMPLE MODEL: ANALYTICAL FORMULAE FOR $f_{\lambda\lambda'}$

For sake of simplicity, we will consider proton knockout from the $s\frac{1}{2}$ shell. No effects will then be produced by the spin-orbit interaction in the final state, but rather from FSI based on the simple geometrical arguments mentioned in the previous Section.

In PWIA the scattering amplitude for the knockout from the $s\frac{1}{2}$ shell in configuration space reads

$$J^{\lambda}_{00\frac{1}{2}0s's}(\mathbf{q}) = \sum_{\tilde{s}} \int \mathrm{d}\mathbf{r}\mathrm{d}\sigma \; \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}} \mathrm{e}^{-\mathrm{i}\boldsymbol{p}'\cdot\boldsymbol{r}} \; \delta_{s'\tilde{s}} \; \langle \tilde{s} \mid \hat{J}_\lambda(\mathbf{q},\mathbf{r},\sigma) \mid s \rangle \; R_{00\frac{1}{2}}(r) Y_{00}(\Omega_{\boldsymbol{r}}) \tag{7}$$

and in momentum space it becomes

$$\begin{aligned} J^{\lambda}_{00\frac{1}{2}0s's}(\mathbf{q}) &= \sum_{\tilde{s}} \int \mathrm{d}\mathbf{p}\mathrm{d}\sigma \; \delta(\mathbf{p}' - \mathbf{p} - \mathbf{q}) \; \delta_{s'\tilde{s}} \; \langle \tilde{s} \mid \hat{J}_\lambda(\mathbf{q},\mathbf{p},\sigma) \mid s \rangle \; R_{00\frac{1}{2}}(p) Y_{00}(\Omega_{\boldsymbol{p}}) \\ &= \langle s' \mid \hat{J}_\lambda(\mathbf{q},\mathbf{p}' - \mathbf{q},\sigma) \mid s \rangle \; \frac{1}{\sqrt{4\pi}} R_{00\frac{1}{2}}\left(|\mathbf{p}' - \mathbf{q}|\right) , \end{aligned} \tag{8}$$

where $\hat{J}_\lambda(\mathbf{q},\mathbf{p},\sigma)$ and $R_{00\frac{1}{2}}(p)$ are the Fourier transforms of the representation in configuration space of the corresponding current operator and radial bound state in Eq. (7), respectively.

The eikonal approximation of the scattering state at the lowest order can be represented by a plane wave with a complex momentum $\mathbf{P}' = \mathbf{p}' + \mathrm{i}\mathbf{p}''$:

$$\mathrm{e}^{-\boldsymbol{p}''\cdot\boldsymbol{R}}\ \mathrm{e}^{\mathrm{i}\boldsymbol{P}'\cdot\boldsymbol{r}} = \mathrm{e}^{-\boldsymbol{p}''\cdot\boldsymbol{R}}\ \mathrm{e}^{\mathrm{i}\boldsymbol{p}'\cdot\boldsymbol{r}}\ \mathrm{e}^{-\boldsymbol{p}''\cdot\boldsymbol{r}}\ , \tag{9}$$

where $\mathbf{R}$ is a constant vector with modulus equal to the nuclear radius. The factor $\mathrm{e}^{-\boldsymbol{p}''\cdot\boldsymbol{R}}$ represents the proper normalization. In fact, if $\hat{z}$ is the propagation axis, the wave enters the nucleus at $\mathbf{r} = -\mathbf{R} \equiv (0, 0, -R)$ with unitary modulus and leaves it at $\mathbf{r} = \mathbf{R} \equiv (0, 0, R)$ with the damping $\mathrm{e}^{-2\boldsymbol{p}''\cdot\boldsymbol{R}}$.

By analytically extending the integrand of Eq. (8) into the complex plane $\mathbf{P}$, it is possible to go beyond the PWIA and, at the same time, to perform the integration still analytically. The extension to the complex plane has two requirements. Firstly, a new definition of the distribution δ of a complex variable (see Appendix), which automatically connects it to the "plane" wave $\mathrm{e}^{\mathrm{i}\boldsymbol{P}'\cdot\boldsymbol{r}}$ of Eq. (9) in the same way as for the case of a real momentum. Secondly, the functions $\hat{J}_\lambda(\mathbf{q}, \mathbf{P}, \sigma), R_{00\frac{1}{2}}(P)$ must be well behaved and their product must asymptotically vanish for $P \to \infty$. With these restrictions Eq. (8) can be extended into the complex plane, i.e.

$$\begin{aligned} J^{\lambda}_{00\frac{1}{2}0s's}(\mathbf{q}) &= \sum_{\tilde{s}} \int \mathrm{d}\mathbf{P}\mathrm{d}\sigma\ \delta(\mathbf{P}' - \mathbf{P} - \mathbf{q})\ \mathrm{e}^{\mathrm{i}(\boldsymbol{P}+\boldsymbol{q}-\boldsymbol{p}')\cdot\boldsymbol{R}}\ \delta_{s'\tilde{s}}\ \langle \tilde{s} \mid \hat{J}_\lambda(\mathbf{q}, \mathbf{P}, \sigma) \mid s\rangle \frac{1}{\sqrt{4\pi}} R_{00\frac{1}{2}}(P) \\ &= \langle s' \mid \hat{J}_\lambda(\mathbf{q}, \mathbf{P}' - \mathbf{q}, \sigma) \mid s\rangle\ \frac{1}{\sqrt{4\pi}} R_{00\frac{1}{2}}\left(|\mathbf{P}' - \mathbf{q}|\right)\ , \end{aligned} \tag{10}$$

where the normalization factor $\mathrm{e}^{-\boldsymbol{p}''\cdot\boldsymbol{R}}$ has been included in a redefinition of the bound state $R_{00\frac{1}{2}}$.

The scattering wave of Eq. (9) represents a simple plane wave damped by an exponential factor driven by $\mathrm{Im}(\mathbf{P}') = \mathbf{p}''$. This corresponds to solve the Schrödinger equation with a complex potential for a particle travelling through homogeneous nuclear matter, i.e.

$$\left(\frac{-\hbar^2}{2m}\nabla^2 + \hat{V} + \mathrm{i}\hat{W}\right)\chi = E\chi\ , \tag{11}$$

or, equivalently,

$$\left(E - \hat{V} - \mathrm{i}\hat{W}\right)\chi = \frac{\hat{\mathbf{P}}' \cdot \hat{\mathbf{P}}'}{2m} = \left(\frac{\hat{\mathbf{p}}'^2 - \hat{\mathbf{p}}''^2}{2m} + \mathrm{i}\ \frac{\hat{\mathbf{p}}' \cdot \hat{\mathbf{p}}''}{m}\right)\chi\ , \tag{12}$$

from which a natural relationship between p'' and the absorbitive part W of the potential is deduced. If the outgoing proton is sufficiently energetic, i.e. $p' \gtrsim 1$ GeV/c, and comes from a bound state with a momentum below the Fermi surface, this approximation has been shown to give reliable results [26,21,27] with a constant $p'' \propto W/p'$. Therefore, the question is whether the description of FSI by a simple plane wave with a constant damping is sufficient to generate an asymmetry in the cross section with respect to the incoming electron helicity, i.e. a nonvanishing fifth structure function. The answer is positive and analytical formulae will be given in the following.

If the damping of the plane wave is constant not only in size, but also in its direction, i.e. $\mathbf{p}'' \parallel \mathbf{p}'$, then

$$\begin{aligned} \mathbf{P}' &= \mathbf{p}' + \mathrm{i}\mathbf{p}'' = \mathbf{p}' + \mathrm{i}\frac{p''}{p'}\mathbf{p}' = \mathbf{p}'\left(1 + \mathrm{i}\frac{p''}{p'}\right) \\ &\equiv (\mathbf{p}_m + \mathbf{q})\left(1 + \mathrm{i}\frac{p''}{p'}\right) , \end{aligned} \tag{13}$$

where $\mathbf{p}_m$ is the missing momentum of the reaction. By substituting Eqs. (10), (13) in Eq. (5) for the $s\frac{1}{2}$ knockout shell, the analytical expression for f'_{01} becomes

$$\begin{aligned} f'_{01} = &-\frac{\sqrt{2}q}{\pi Q}\; R_{00\frac{1}{2}}\left(|\mathbf{p}_m + \mathrm{i}\mathbf{p}''|\right) p_{m_x}\frac{p''}{p'} \\ &\left[S(\omega, q^2) + D(\omega, q^2, p'^2, p''^2) + D'(\omega, q)(p_{m_z} + q)\right] , \end{aligned} \tag{14}$$

where

$$\begin{aligned} D(\omega, q^2, p'^2, p''^2) &= \frac{3\kappa F_1 F_2}{8m^3}Q^2 + \frac{F_1^2}{m}\left[\frac{1}{4m^2}\left(2p'^2 + 2p''^2 + q^2 + \frac{1}{2}Q^2\right) + \frac{5}{64m^3}\omega q^2 - 1\right] , \\ D'(\omega, q) &= -\frac{F_1^2}{2m^3}q\left(1 + \frac{5}{8m}\omega\right) \end{aligned} \tag{15}$$

come from the part of the current operator $\hat{J}_\lambda(\mathbf{q}, \mathbf{P}' - \mathbf{q}, \sigma)$ which does not flip the initial nucleon spin, while

$$S(\omega, q^2) = \frac{F_1 + 2\kappa F_2}{8m^3}q^2\left[F_1 + \kappa F_2 + \frac{\omega}{4m}(F_1 + 2\kappa F_2)\right] \tag{16}$$

is produced by the spin-flip part. Here, the vector components along $\hat{x}$, $\hat{z}$ are referred to the hadronic plane $(\mathbf{p}', \mathbf{q})$. If $\alpha = 0^{\mathrm{o}}$ the latter coincides with the scattering plane and

p_{m_x} actually represents the component along the $\hat{x}$ axis of the lab system described in Fig. 1. If $\alpha = 90^o$ the hadronic plane is perpendicular to the scattering plane and p_{m_x} refers to the component along the $\hat{y}$ axis of the lab system. No ambiguity should arise from the interpretation of the components along the $\hat{z}$ axis which always points in the direction of $\mathbf{q}$.

If the experimental setup is such that the spectrometer of the hadron arm detects the outgoing protons on a plane perpendicular to the scattering plane, i.e. for $\alpha = 90^o$ in Fig. 1, from Eqs. (1), (6) the helicity asymmetry takes the simple form

$$A = \frac{\rho'_{01} f'_{01}}{\rho_{00} f_{00} + \rho_{11} f_{11} - \rho_{1-1} f_{1-1}} \quad \underset{Q\to\infty}{\longrightarrow} \quad \frac{\rho'_{01} f'_{01}}{\rho_{11} f_{11}} \,, \tag{17}$$

because for increasing Q the nuclear response becomes more and more transverse. Since, analogously to Eq. (5), the structure function f_{11} is given in terms of the scattering amplitudes as [4]

$$f_{11} = \sum_{s' m_l s \overline{m_l} \overline{s}} \left(l \tfrac{1}{2} m_l s | j m_j\right) \left(l \tfrac{1}{2} \overline{m_l} \overline{s} | j m_j\right) \; \{ J^{1}_{nljm_l s' s} J^{1\,*}_{nlj\overline{m_l} s' \overline{s}} + J^{-1}_{nljm_l s' s} J^{-1\,*}_{nlj\overline{m_l} s' \overline{s}} \} \tag{18}$$

and the components of the lepton tensor read [4]

$$\rho'_{01} = \frac{Q^2}{q^2} \frac{1}{\sqrt{2}} \tan\frac{\theta}{2} \,, \qquad \qquad \rho_{11} = \frac{Q^2}{2q^2} + \tan^2\frac{\theta}{2} \,, \tag{19}$$

the final analytical expression of the helicity asymmetry for the knockout from the $s\frac{1}{2}$ shell is

$$\begin{aligned} A = & -\frac{4qQ\tan\frac{\theta}{2}}{Q^2 + 2q^2\tan^2\frac{\theta}{2}} \; p_{m_x} \frac{p''}{p'} \; \left[S(\omega, q^2) + D(\omega, q^2, p'^2, p''^2) + D'(\omega, q)(p_{m_z} + q) \right] \\ & \times \Big[\overline{S}(\omega, q^2, p'^2, p''^2) + \overline{S}'(\omega, q)(p_{m_z} + q) + \overline{S}''(\omega^2, p'^2, p''^2)(p_{m_z} + q)^2 \\ & \quad + \overline{D}(\omega^2, q^2, p'^2, p''^2)(p^2_{m_x} + p^2_{m_y}) + \overline{D}'(q, p'^2, p''^2)(p_{m_z} + q)(p^2_{m_x} + p^2_{m_y}) \Big]^{-1} \,. \end{aligned} \tag{20}$$

Analogously to Eqs. (15), (16) the functions $\overline{S}, \overline{S}', \overline{S}''$ and $\overline{D}, \overline{D}'$ are produced by the spin flipping and non spin flipping parts of the interaction, respectively. Their expressions are

$$\begin{aligned} \overline{S}(\omega, q^2, p'^2, p''^2) = & \left(\frac{F_1 + 2\kappa F_2}{4m^2} \omega \right)^2 (p'^2 + p''^2) + \frac{q^2}{2m^2} \left[F_1 + \kappa F_2 + \frac{F_1 + 2\kappa F_2}{4m} \omega \right]^2 \\ & - \frac{F_1 + \kappa F_2}{8m^4} q^2 \left[4F_1(p'^2 + p''^2) + 2F_1 q^2 + \kappa F_2 Q^2 \right] \,, \end{aligned}$$

$$
\begin{aligned}
\overline{S}'(\omega, q) &= \frac{q}{2m^3}\left[\frac{F_1}{m}(F_1+\kappa F_2)q^2 - (F_1+\kappa F_2)(F_1+2\kappa F_2)\omega - \frac{(F_1+2\kappa F_2)^2}{4m}\omega^2\right] , \\
\overline{S}''(\omega^2, p'^2, p''^2) &= \left(\frac{F_1+2\kappa F_2}{4m^2}\omega\right)^2 \left(1+\frac{p''^2}{p'^2}\right) , \\
\overline{D}(\omega^2, q^2, p'^2, p''^2) &= \frac{F_1}{m^2}\left(1+\frac{p''^2}{p'^2}\right)\left[F_1 - \frac{F_1}{m^2}(p'^2+p''^2) - \frac{F_1}{2m^2}q^2 - \frac{\kappa F_2}{4m^2}Q^2\right] , \\
\overline{D}'(q, p'^2, p''^2) &= \frac{F_1^2}{m^4} q \left(1+\frac{p''^2}{p'^2}\right) .
\end{aligned} \tag{21}
$$

Finally, it should be noticed, from Eqs. (14), (20) respectively, that both f'_{01} and A depend on p''/p', which is related to the imaginary part of the complex momentum $\mathbf{P}'$ defined in Eq. (13). This ratio gives a measure of the damping of the scattering wave, i.e. of the FSI. In fact, for $p'' \to 0$ the damping vanishes: the scattering wave becomes a plane wave with momentum $\mathbf{P}' \equiv \mathbf{p}'$ and f'_{01}, A vanish, as it must be in PWIA.

IV. RESULTS

In this Section results will be shown for the fifth structure function f'_{01} of Eq. (14) and for the helicity asymmetry A of Eq. (20) for the proton knockout from a $s\frac{1}{2}$ shell by a polarized electron beam. For sake of consistency with previous calculations [26,21,27] and available measurements [8,28], the ^{12}C target has been selected. The choice of the residual ^{11}B with quantum numbers $s\frac{1}{2}$ is justified, as mentioned in the previous Section, by the absence of any FSI due to spin-orbit effects, which not only makes formulae simpler, but also clarifies the pure geometrical nature of FSI introduced by a plane wave with complex wave vector. Because of the purely absorbitive nature of the damping, the energy range available to the final proton has been selected above the inelastic threshold of $p' \sim 1$ GeV$/c$.

It has been shown elsewhere [27] that the nuclear response for p_m well above the target Fermi momentum $p_{\rm Fermi}$ is dominated by FSI with a nontrivial structure, while for $p_m \lesssim p_{\rm Fermi}$ it can be described as the PWIA contribution with an additional constant damping. Since the propagation of the outgoing proton with a complex wave vector can actually be pictured as a plane wave with a constant damping, it seems natural to select values of p_m

inside a range where the adopted representation of FSI is not too inadequate. The Fermi momentum of ^{12}C is $p_{\rm Fermi} = 221$ MeV/c; therefore, in the following, a typical value of $p_m = 200$ MeV/c will be used.

In the considered domain of inelastic processes $p' \sim q \gg p_m$. Therefore, the kinematics is almost purely transverse. In the following, without loss in generality, it will be kept exactly transverse, i.e. with $p_{m_z} = 0, p_{m_x} = 200$ MeV/c. The damping factor p'' has been shown, in the previous Section, to be directly related to the the imaginary part W of the equivalent optical potential. If the Glauber choice of $W \propto p'$ is adopted, the observed damping in the NE18 data is reproduced by selecting $V = 0, W = 0.036\ p'$ MeV [21,27]. Correspondingly, in the following $p'' = 50$ MeV/c will be used.

As a cross-check, the helicity asymmetry of Eq. (20) for the present choices of $\mathbf{p}_m$ and p'' has been compared in the range $0.6 \leq q \leq 1$ GeV/c and for a quasielastic kinematics ($\omega \simeq q^2/2m$) with the output of the numerical code `PV5FF` developed in Pavia, which successfully describes the amount of presently available $(e, e'p)$ and $(\vec{e}, e'p)$ data at medium proton energies in the framework of DWIA and including spin-dependent FSI and Coulomb distortion of the electron waves [4]. In Fig. 2 the solid line corresponds to the analytical formula, while the dashed line is the numerical result obtained with the complex optical potential $V = 0, W = 0.036\ p'$ MeV and the bound state of Comfort and Karp [29] for the $s\frac{1}{2}$ shell of ^{12}C. The agreement is satisfactory for $q \lesssim 0.8$ GeV/c, while above this threshold the accuracy required by the delicate cancellations taking place in the numerator of Eq. (17) is not fulfilled by the numerical code, which was optimized for lower energies.

A. The fifth structure function f'_{01}

In Eq. (14) emphasys has been put on identifying the single contributions coming from different reaction mechanisms (flipping or non flipping of the nucleon spin) to put in better evidence the delicate interplay that leads to a very small structure function.

In Fig. 3 the f'_{01} (apart from the constant factor $-\sqrt{2} R^2_{00\frac{1}{2}} \left(|\mathbf{p}_m + \mathrm{i}\mathbf{p}''| \right) / \pi$) is represented

by the solid line for the ${}^{12}\mathrm{C}(\vec{e},e'p)^{11}\mathrm{B}_{s\frac{1}{2}}$ reaction as a function of q with $p_{m_x} = 200$ MeV/c and $p'' = 50$ MeV/c. The results for the functions $D(\omega, q^2, p'^2, p''^2)$, $D'(\omega, q)$, $S(\omega, q^2)$ of Eqs. (15), (16) are indicated by the short-dashed, long-dashed and dot-dashed lines, respectively. It should be noticed that the total result is amplified by a factor 10^2 with respect to each addendum. This dramatic cancellation is the natural counterpart of f'_{01} being defined as the difference of contributions coming from the interference between longitudinal ($\lambda = 0$) and transverse ($\lambda = \pm 1$) components of the nuclear current (see Eq. (5)). This peculiar feature on one side makes f'_{01} very interesting because extremely sensitive to reaction channels emphasized in the interference (to FSI, in this case of quasielastic knockout), but on the other side produces a very small, hardly measurable quantity.

Experimentally, it is possible to directly determine f'_{01} by performing an absolute measurement of the corresponding unpolarized cross section Σ and of the helicity asymmetry A [8]. The knowledge of Σ and A in Eq. (6) makes it possible to isolate the helicity dependent part of the cross section, Δ, and consequently the fifth structure function through the relation

$$f'_{01} = A\Sigma \, \frac{8\pi^2 Q^4 p_{\mathrm{e}} p'_{\mathrm{e}}}{e^4 \rho'_{01}} \, . \tag{22}$$

B. The helicity asymmetry A

In Fig. 4a the helicity asymmetry A of Eq. (20) is plotted as a function of q for the ${}^{12}\mathrm{C}(\vec{e},e'p)^{11}\mathrm{B}_{s\frac{1}{2}}$ reaction with $p_{m_x} = 200$ MeV/c, $p'' = 50$ MeV/c and $\theta = 40^{\mathrm{o}}$. Again, because of the cancellations occurring inside f'_{01} the asymmetry quickly becomes very small. A zoom of it is given in Fig. 4b, which shows an interesting structure with a change of sign and a long asymptotic tail. Despite the fact that the asymmetry measurement is an experimentally favourite situation, the absolute size of A is probably too small to be ever detected.

However, it is interesting to study the asymptotic behaviour of this smooth dependence upon q, or equivalently Q. It has already been mentioned that for increasing Q the response

to an electron probe is known to become more and more transverse with respect to the helicity of the virtual photon exchanged. In pQCD simple dimensional arguments [30] show that for exclusive processes like elastic electron-proton scattering the ratio between the Dirac and Pauli proton form factors, F_1/F_2, scales as Q^2. At the cross section level this corresponds to the linear scaling in $1/Q$ of the ratio $J^0/J^{\pm 1}$, where J^0 ($J^{\pm 1}$) is the helicity amplitude for absorption by a proton of a longitudinally (transversely) polarized photon. Apart from kinematical factors, the fifth structure function f'_{01} is approximately a linear combination of products $J^0 J^{\pm 1}$, while the dominant purely transverse structure function f_{11} is essentially given by $(J^{\pm 1})^2$. Therefore, one would naively deduce from Eq. (17) that the helicity asymmetry itself asymptotically scales as $1/Q$.

But the f'_{01} is not just a linear combination of products $J^0 J^{\pm 1}$, as it is evident from Eq. (5). The cancellations between contributions of the same order in powers of $1/Q$ are very sensitive to the relativistic corrections to the current operator and produce a nontrivial scaling law. Assuming that for large Q^2 the Bjorken variable $x = Q^2/2m\omega$ is approximately constant and, consequently, $\omega \sim q \sim Q^2$, from Eqs. (4), (15), (16) and (21) it can be deduced that the helicity asymmetry of Eq. (20) scales as

$$A \underset{Q\to\infty}{\sim} \frac{1}{Q^5} \ . \tag{23}$$

In the energy domain pertinent to the planned ELFE setup [12], the previous assumptions on x, q/Q^2, ω/Q^2 do not hold yet. A different tail as power of $1/Q$ must be expected for A. In fact, in Fig. 5 the helicity asymmetry is shown, multiplied by Q^4, as a function of q for the same reaction and in the same kinematical conditions as in the previous figure. The plateau indicates that in this energy window the scaling is different from what is predicted by Eq. (23), or, in other words, that the asymptotic behaviour is not yet reached within the present nonrelativistic reduction of the current operator at the order described in Eq. (3) (see also Ref. [31]).

Finally, since the kinematics is here purely transverse and for high p', q the longitudinal component of the missing momentum, p_{m_z}, is anyway small, the asymmetry is approximately

linearly dependent on p_{m_x} ($p_{m_y} = 0$) and, consequently, does not show any interesting structure with respect to p_{m_z}.

V. CONCLUSIONS

The ${}^{12}\mathrm{C}(\vec{e}, e'p)^{11}\mathrm{B}_{s\frac{1}{2}}$ reaction has been analyzed assuming for the scattering state a plane wave with complex wave vector. This choice allows for obtaining analytical formulae for the different components of the nuclear response; it corresponds to the situation where the outgoing proton emerges as a free particle but its wave function is exponentially damped with a rate related to the imaginary part p'' of the complex wave vector. This picture is also equivalent to solve the problem for the scattering state in the lowest-order eikonal approximation or, alternatively, to compute the FSI of the outgoing proton travelling across absorbitive homogeneous nuclear matter represented by a complex potential. In fact, p'' has been shown to be directly related to the imaginary part of this potential and is of the same order of magnitude.

Since the residual nucleus is left with quantum numbers $s\frac{1}{2}$, there are no FSI due to spin-orbit effects. The modification of the emerging plane wave is the only reason why the symmetry between protons emitted above and below the scattering plane is broken. The different path followed on the way out of the nucleus makes them have different rescatterings with the residual and, consequently, produces a phase difference with respect to the channel where they are knocked out directly as free particles. In these conditions, and in general whenever there are at least two predominant reaction channels with different phases [7], the cross section part depending on the electron helicity h does not vanish.

In particular, the fifth structure function f'_{01} can be used to disentangle interfering processes and, in the present case of quasielastic kinematics, to study the rescattering amplitudes. The f'_{01} has been analyzed for the previously mentioned reaction in purely transverse kinematics for the energy range above the inelastic threshold ($p' \sim q \gtrsim 1$ GeV/c), where the FSI are almost purely absorbitive and the eikonal approximation is known to be reli-

able [26,21,27]. Inspection of the analytical formula shows that dramatic cancellations take place among the different contributions coming from the (non) spin-flipping parts of the interaction current. As a result, the absolute size of f'_{01} is very small and, presumably, hardly observable.

However, the asymmetry between particles detected above and below the scattering plane is equivalent to the asymmetry for particles emitted in the same direction but for opposite h. The helicity dependent cross section Δ, proportional to f'_{01}, can be singled out by making coincidence measurements with a fixed spectrometer at an angle out of scattering plane and flipping the helicity of the incoming polarized electron. High precision data can be obtained with this asymmetry technique, because most systematic errors cancel in the ratio [8].

The analytical formula for the helicity asymmetry A has been studied in the same previous kinematics, specifically focussing on its asymptotic behaviour for very large energy and momentum transfer. In fact, despite of its very small absolute size, it shows an interesting structure with a change of sign and a long asymptotic tail.

The occurrence inside f'_{01} of cancellations between competing contributions, asymptotically scaling with the same power of $1/Q$, is very sensitive to higher-order relativistic corrections to the current operator and produces in the related A a nontrivial scaling law for large Q, which cannot be naively deduced from dimensional arguments applied to the elementary photo-quark reaction amplitudes [30]. Moreover, this asymptotic scaling occurs for very large values of Q outside the range available to the operational or planned setups of modern electron accelerators, such as CEBAF or ELFE. In particular, in the energy domain of the latter [12] the asymmetry A still shows a scaling behaviour, but with a power law in $1/Q$ which is different from the asymptotic one.

In summary, within the present nonrelativistic reduction of the current operator at the order described in Eq. (3), the helicity asymmetry is very small in its absolute size but shows a long nontrivial tail for large Q. For $Q \to \infty$ it scales as $1/Q^5$, but it approaches the asymptotic regime very slowly, even locally showing, for large but finite Q, different scaling behaviours.

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APPENDIX:

The usual distribution δ can be defined as

$$\delta(x-\overline{x}) = \lim_{\varepsilon\to 0} \frac{\varepsilon}{(x-\overline{x})^2+\varepsilon^2} = \lim_{\varepsilon\to 0} \frac{1}{2\pi\mathrm{i}} \left\{ \frac{1}{x-\overline{x}-\mathrm{i}\varepsilon} - \frac{1}{x-\overline{x}+\mathrm{i}\varepsilon} \right\} , \tag{A1}$$

where $x, \overline{x} \in \mathbb{R}$. This definition can be generalized to the case of the distribution δ of the complex variable z as [32]

$$\delta(z-\overline{z}) = \lim_{\varepsilon\to 0} \frac{1}{2\pi\mathrm{i}} \left\{ \frac{1}{z-\overline{z}-\mathrm{i}\varepsilon} - \frac{1}{z-\overline{z}+\mathrm{i}\varepsilon} \right\} . \tag{A2}$$

The new definition of Eq. (A2) keeps the usual properties of the δ, in particular

$$\int_C \mathrm{d}z\ \delta(z-\overline{z}) f(z) = f(\overline{z}) , \tag{A3}$$

where C is a integration path on the complex plane, extending to $\mathrm{Re}(z) \to \pm\infty$ on the real axis but going through the point $z = \overline{z}$, and $f(z)$ is an analytical complex function with the property $f(z) \to 0$ for $|z| \to \infty,\ \mathrm{Im}(z) > 0$ ($\mathrm{Im}(z) < 0$) if C is closed in the upper (lower) part of the complex plane.

From Eq. (A3) it follows that

$$\int_C \mathrm{d}z\ \delta(z-\overline{z}) \mathrm{e}^{\mathrm{i}xz} = \mathrm{e}^{\mathrm{i}x\overline{z}} , \tag{A4}$$

which generalizes the standard relationship between the δ and the plane wave through the Fourier transformation. Eq. (A4) can be demonstrated by closing the path C with a semicircle in the upper part of the complex plane ($|z| \to \infty,\ \mathrm{Im}(z) > 0$) for $x \geq 0$ or in the lower part ($|z| \to \infty,\ \mathrm{Im}(z) < 0$) for $x < 0$.

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FIGURES

FIG. 1. The kinematics for the one-nucleon knockout process from a polarized electron beam.

FIG. 2. The helicity asymmetry A, multiplied by 10^3, as a function of the momentum transfer q in the range $0.6 \leq q \leq 1$ GeV/c for the $^{12}\mathrm{C}(\vec{e}, e'p)^{11}\mathrm{B}_{s\frac{1}{2}}$ reaction in quasielastic completely transverse kinematics ($p_m \equiv p_{m_x} = 200$ MeV/c) at the scattering angle $\theta = 40^{\mathrm{o}}$. The solid line is the outcome of the analytical formula of Eq. (20) with the damping $p'' = 50$ MeV/c (see text). The dashed line is the output of the numerical code `PV5FF` based on the DWIA with a purely imaginary optical potential with depth $W = 0.036\, p'$ MeV and with the bound state obtained from the potential of Comfort and Karp (see text).

FIG. 3. The solid line is the fifth structure function f'_{01} of Eq. (14), multiplied by 10^2 and divided by $-\sqrt{2}R^2_{001/2}/\pi$, as a function of the momentum transfer q for the $^{12}\mathrm{C}(\vec{e}, e'p)^{11}\mathrm{B}_{s\frac{1}{2}}$ reaction in the same kinematical conditions as for the solid line in Fig. 2. The short-, long- and dot-dashed lines are the $D(\omega, q^2, p'^2, p''^2), D'(\omega, q)$ and $S(\omega, q^2)$ functions of Eqs. (15), (16), respectively.

FIG. 4. The helicity asymmetry A, multiplied by 10^3, as a function of the momentum transfer q for the $^{12}\mathrm{C}(\vec{e}, e'p)^{11}\mathrm{B}_{s\frac{1}{2}}$ reaction in the same kinematical conditions as for the solid line in Fig. 2. Upper part (a) for the range $1.4 \leq q \leq 4.5$ GeV/c, lower part (b) for the range $3.5 \leq q \leq 10$ GeV/c and in an amplified scale.

FIG. 5. The product $A * Q^4$, multiplied by 10^3, as a function of the momentum transfer q in the range $5 \leq q \leq 23$ GeV/c for the $^{12}\mathrm{C}(\vec{e}, e'p)^{11}\mathrm{B}_{s\frac{1}{2}}$ reaction in the same kinematical conditions as for the solid line in Fig. 2.

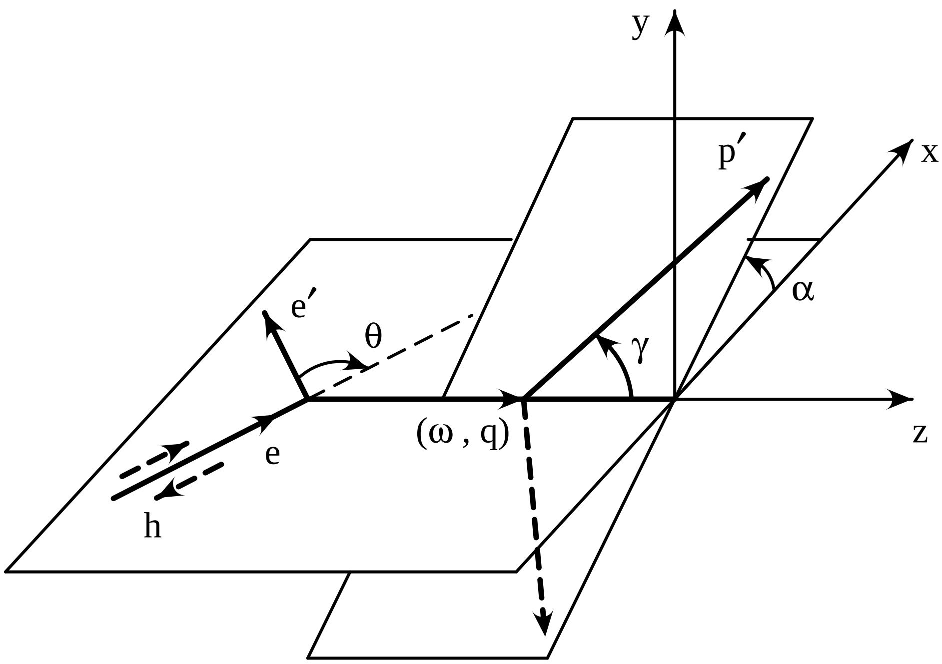

Fig. 1

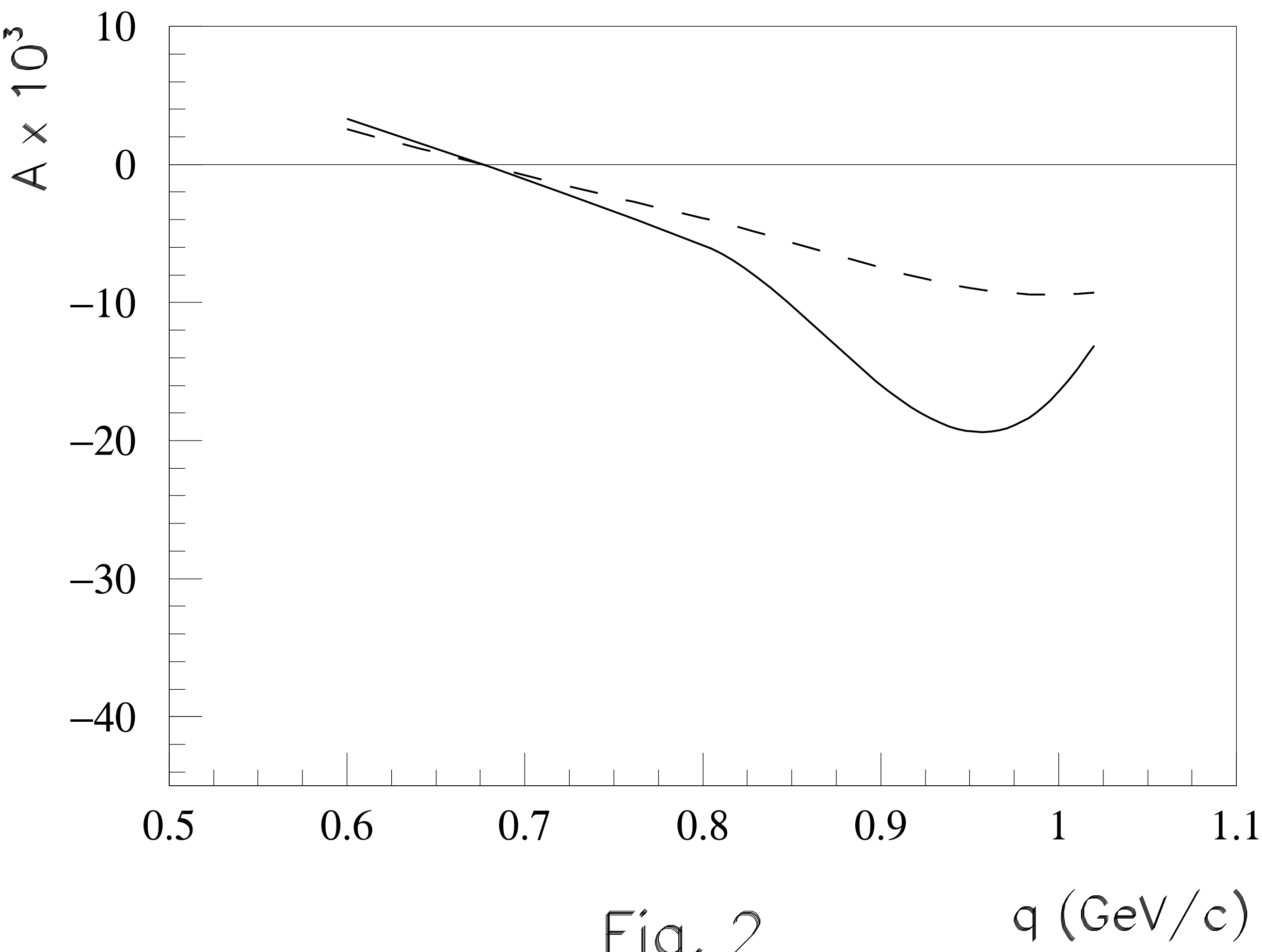

Fig. 2

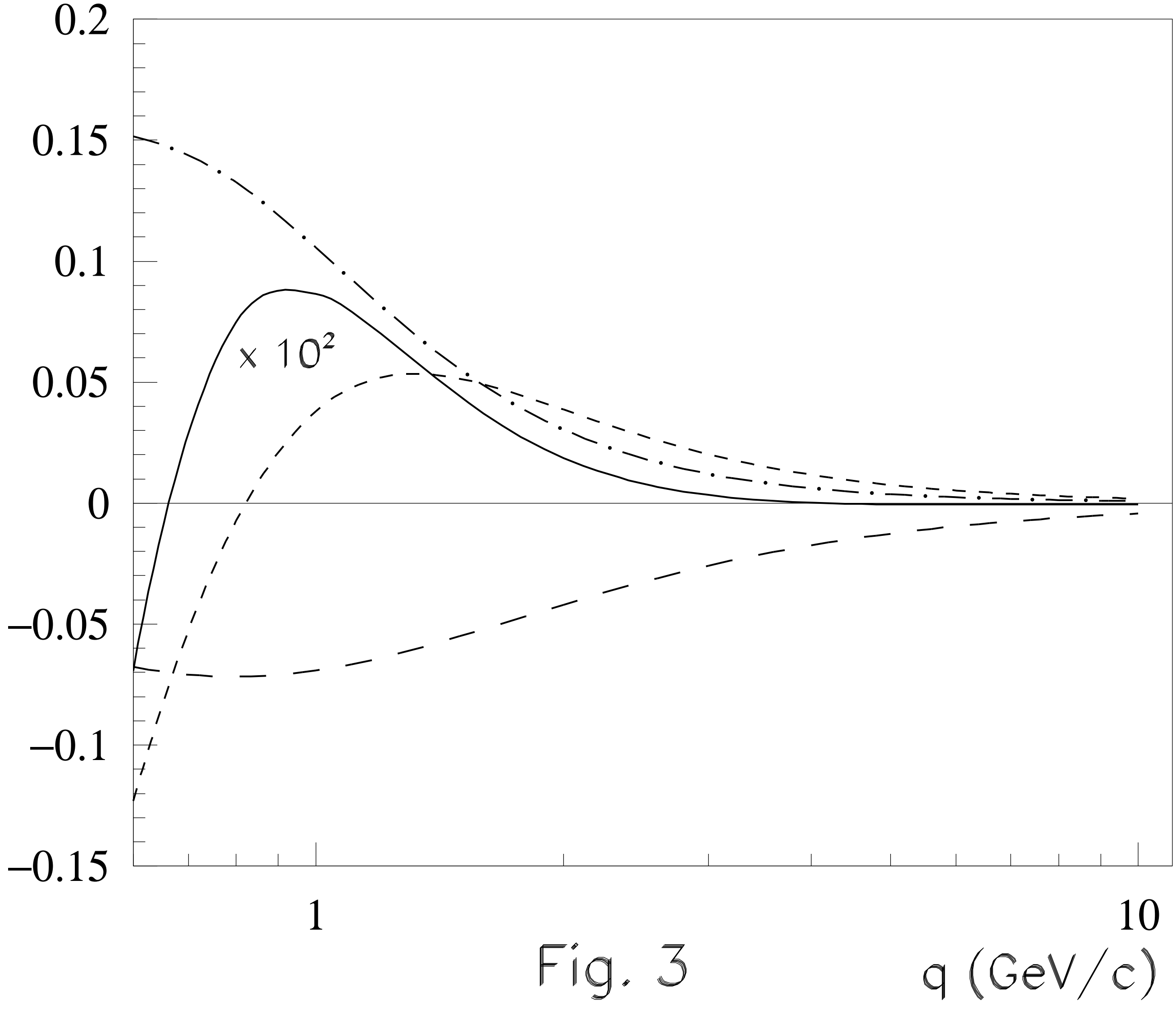

Fig. 3

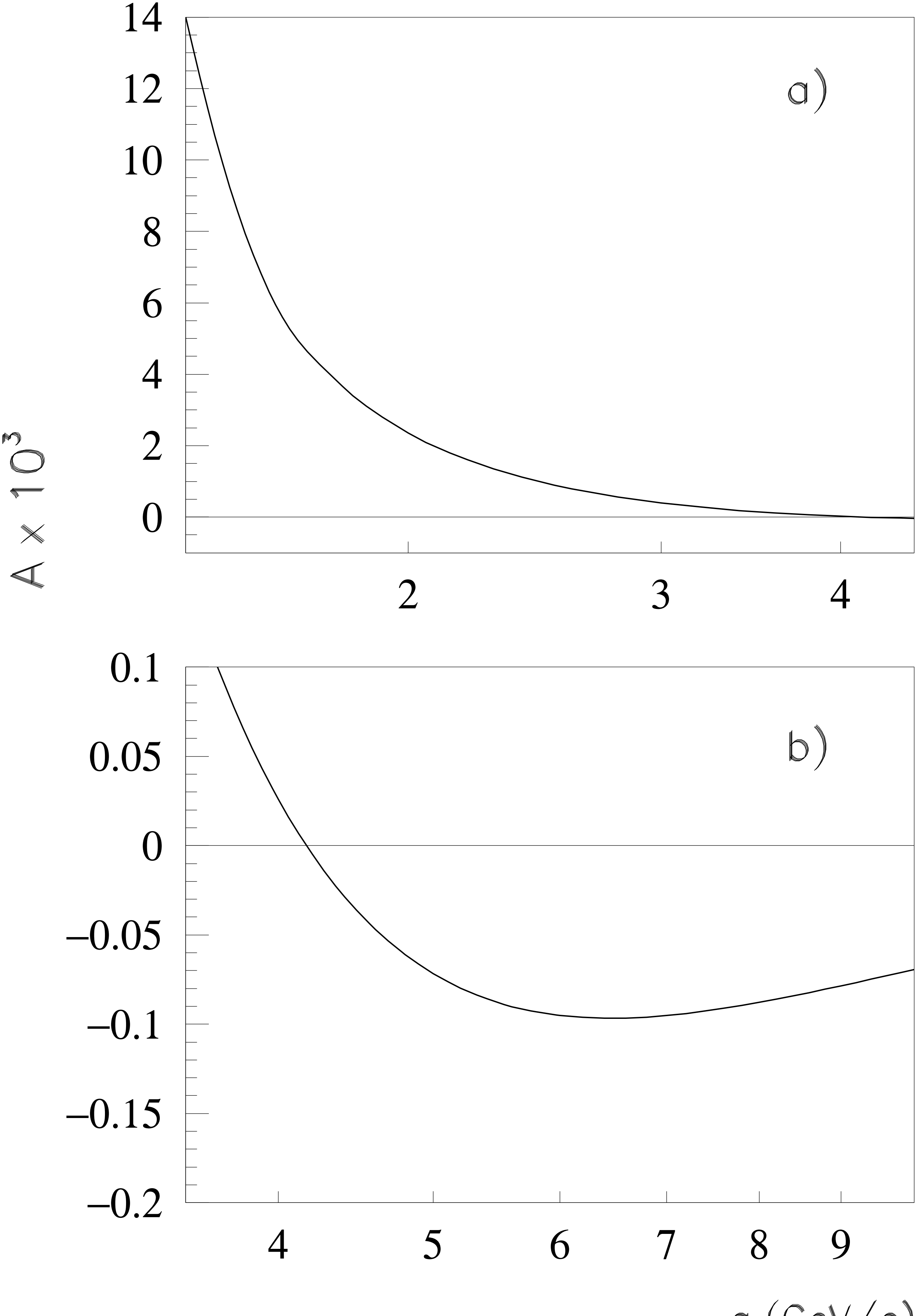

Fig. 4

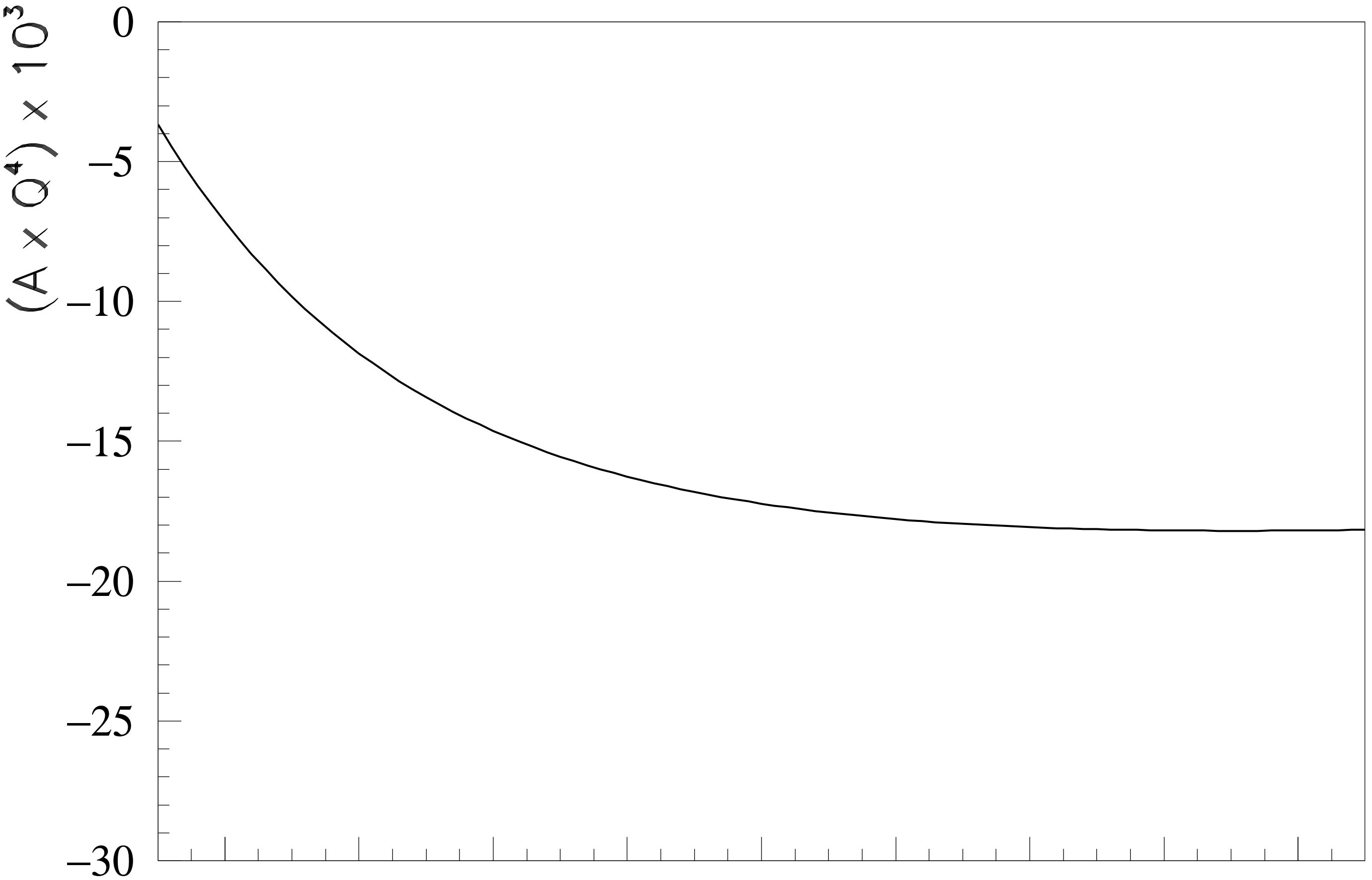

Fig. 5